Coevolutionary Dynamics in a Minimal Substrate

Richard A. Watson **Jordan B. Pollack**

Dynamical and Evolutionary Machine Organization,
Volen Center for Complex Systems, MS018, Brandeis University,
Waltham, MA 02454, USA
richardw@cs.brandeis.edu

Abstract

One of the central difficulties of coevolutionary methods arises from 'intransitive superiority' – in a two-player game, for example, the fact that *A* beats *B*, and *B* beats *C*, does not exclude the possibility that *C* beats *A*. Such cyclic superiority in a coevolutionary substrate is hypothesized to cause cycles in the dynamics of the population such that it 'chases its own tail' - traveling through some part of strategy space more than once despite apparent improvement with each step. It is often difficult to know whether an application domain contains such difficulties and to verify this hypothesis in the failure of a given coevolutionary set-up. In this paper we wish to elucidate some of the issues and concepts in an abstract domain where the dynamics of coevolution can be studied simply and directly. We define three simple 'number games' that illustrate intransitive superiority and resultant oscillatory dynamics, as well as some other relevant concepts. These include the distinction between a player's perceived performance and performance with respect to an external metric, and the significance of strategies with a multi-dimensional nature. These features alone can also cause oscillatory behavior and coevolutionary failure.

Keywords: Coevolution, intransitive superiority, multiple dimensions, coevolutionary failure.

1 INTRODUCTION

Coevolution has become increasingly popular in Evolutionary Algorithms research (e.g. Hillis 1992, Sims 1994, Juille 1996, Miller & Cliff 1994). The basic idea behind the approach seems intuitive enough – rather than evolve individuals against a fixed objective metric, we engage individuals in the task of improving their performance against other evolving individuals. One of the most unequivocal benefits of this approach comes from the fact that for many machine learning domains a suitable objective metric of performance is simply not available. Examples include the coevolution of pursuit and evasion behaviors (Miller and Cliff 1994, Reynolds 1994), and competitive manipulation of physical objects (Sims 1994). Apart from this primary benefit of providing some target for performance, coevolution is commonly understood to have several other benefits. The following list is not a comprehensive account of coevolution's supposed benefits – rather we have selected those ideas for which we will be able to illustrate related issues in our experiments – but, these ideas cover some of those most common in the coevolution literature. We use examples from the domain of chess but the concepts apply equally to any task that can be described using performance with respect to an opponent:

a) Providing a target that is 'hittable' – gradient.
If any two novice chess players play against, say, a Grand Master then both will lose and their performance will be indistinguishable. But if the novices play against each other the superiority of one with respect to the other will be revealed. By engaging players in the mutual pressure to outperform one-another coevolution provides adaptive gradients that might otherwise be hard to engineer. Pollack & Blair (1996) provide an example where 'self-play' provides a gradient for learning.

b) A target that is relevant – focusing.
If we have our two novices play against a (fixed) set of other chess players of various abilities then the number of games they win might be different, and we may select the better. But how are we to devise this set of opponents? A random set may not be representative of all chess players. Any given set may fail to test certain aspects of play. By using other evolving players as opponents coevolution may focus adaptation on those aspects of a task that have not yet been optimized. Examples where one 'species' is used to provide a focused test-set for another species include Hillis (1992), and Juille (1996).

c) A moving target – open-endedness.
Even if we could find a representative fixed set of opponents that provided a gradient from novice play through to master level, any fixed set will have an

upper limit. Using coevolution there is always the potential to be a better player than the best player found so far, and when found such a player provides the new target to beat. Open-endedness is often cited as a benefit of coevolution, e.g. (Ficici 1998)

However, although these notions are common, and seem intuitive enough, they are not very well defined. Moreover, there is increasing awareness, in this same community of researchers, that coevolution can sometimes introduce as many problems as it solves. In the list below we describe informally some of the ways in which a coevolving target for performance might not be hittable, relevant, or moving in the right direction:

a) Loss of gradient.
Suppose an evolving population of opponents becomes too good – we may find ourselves with an 'unhittable' target once more. For example, perhaps evasion is very much easier than pursuit, and none of the evaders can be caught. In this case we lose the gradient information and players may drift without improvement.

b) Focusing on the wrong things.
The ability to focus on an opponent's weakness can provide an easy way to win. This may produce degenerate players that over-specialize on opponents weaknesses, and fail to learn a task in a general way.

c) 'Relativism'.
When opponents co-adapt, and describe a task for one another, we suppose that they will 'leap-frog' one another in steps of increasing performance. But, if *A*'s performance is defined by *B*, and *B*'s performance is defined by *A*, then the adaptive system is disconnected from any absolute measure of performance. Two good players get the same score against each other, as do two bad players. So, supposing variation is equally likely to take the standard of play down as up (perhaps more likely), what is to ensure that these moving targets will move the way we want them to? Such relativism may enable ways for the players to 'subvert' the game we as researchers had in mind, and may lead to mediocre players that never improve.

Problems such as these may be involved in some of the failures in the literature, but it is very difficult to be sure. Identifying the cause of a failure is complicated by the fact that it is very difficult to separate the dynamics of the coevolutionary set-up from the details of the application domain, be it backgammon, robotics, pursuit and evasion, or whatever. Thus, the benefits and the problems with coevolution continue to be a bit vague and ill-defined; often going no further than the level of description we have given above, and relying on metaphors like 'arms race' and 'collusion'.

Some work has addressed issues in coevolution and relativism in the abstract, which enables particular underlying concepts to be illustrated and investigated (Maynard-Smith 1982, Cliff & Miller 1995, Kauffman 1993). In this style, we introduce in this paper a minimal substrate in which coevolutionary concepts, dynamics, and problems can be investigated - in particular, the importance of intransitive superiority. Specifically, we evolve scalar values and vectors under various coevolutionary set-ups. This substrate enables us to illustrate some important concepts that may be underlying the problems we introduced above. Our experiments provide concrete examples for each of the ideas we have discussed, and assist us in gaining some defining concepts that may be useful in diagnosing coevolution failures.

The following sections are organized as follows: Section 2 introduces some of the concepts we see as central to the issues we described above, and describes the minimal substrate we use for our investigations. Section 3 describes other aspects of our coevolutionary set-up. Section 4 describes experiments that each illustrate a different potential cause of failure in coevolution. Section 5 concludes.

2 A MINIMAL SUBSTRATE

In this section we introduce the minimal substrate that we will use in our experiments. In the process we will describe some of the concepts that we see as important for understanding the issues involved in coevolution.

2.1 SCALARS

We commence by considering the coevolution of scalar values. For example, we could evolve integers using coevolutionary techniques to find high values. In this domain we know that the task is trivial and that *evolving* integers is easy, thus any problems we have using *coevolutionary* techniques are a product of coevolution. Although it may seem too trivial to be of use we will see that there are several phenomena that can be illustrated with its help.

We assert that the goal of the evolutionary process is to maximize, a, the value being evolved. Clearly, if we evolve integer values using a fitness function, $f(a)=a$, then the problem is trivial. However, we will investigate what happens when we coevolve these values using a fitness function, $f(a,S)$, that returns a value for one number, a, *with respect to* a set of other numbers, S. S is a *sample* of individuals against which a will be tested. For the purposes of our experiments we will use $f(a,S)$ that counts the number of members of S that are less than a:

$$f(a,S) = \sum_{i=1}^{|S|} score(a,S_i) \qquad \textbf{/eq.1}$$

where $score(a,b)=1$ if $a>b$, 0 otherwise.

In this way we may evolve the scalar values as if they were playing a 'greater than' game, rather than evolving them against an objective fitness function.

Clearly, if S were the complete set of possible values in the domain of a, then our coevolutionary set-up would be

the same as the trivial evolutionary case. But naturally, for our purposes S will consist of other coevolving individuals. In some cases these will be drawn from the same population and may therefore be genetically related. In other cases, they will be drawn from a separate coevolving population. We will see that the effects of this choice can be significant.

2.2 OBJECTIVE FITNESS, AND SUBJECTIVE FITNESS

In evolutionary algorithms the fitness of an individual is given by a 'fitness function' or 'objective function' - this provides some measure of the individual's performance or quality with respect to the task at hand. In coevolution, there is still a fitness function, e.g. Equation 1 - but the value it returns is no longer objective, it is subject to the sample chosen. To make the distinction clear, we will call the metric that we as researchers seek to optimize the *objective fitness*, and we will call the metric of performance as perceived by the co-evolving individual the *subjective fitness*.[1]

We asserted above that our objective fitness was $f(a)=a$. It seems fairly likely that anything adapting under our 'greater than' game will become maximized, as we intended, but this is not necessarily so. Consider making a judgment: which of a and b is to be preferred? (where a and b are any two individuals). With respect to our objective metric, the preferred individual is whichever is larger in value. They may be equally preferred only if they are equal in value. Let us denote this objective preference relation as $P_{obj}(a,b)$. In the coevolutionary game, the one that will be preferred will be whichever gets the highest value when played in the 'greater than' game against S. This is its *subjective* fitness. If $f(a,S_a)$ is greater than $f(b,S_b)$ (according to Equation 1) then a is preferred. Let us write the coevolutionary preference as $P_{subj}(a,b)$, then we have stated that $P_{subj}(a,b)=P_{obj}(f(a,S_a), f(b,S_b))$. Notice that we do not assume that a and b are evaluated against the same S. And it should be clear that we may get a different preference depending on how we choose S_a and S_b.

Many of the problems we encounter in coevolution can be described as arising from the separation between a player's performance as they perceive it, from their performance with respect to an external metric. A mismatch of preference relations from objective and subjective metrics, i.e. $P_{subj}(a,b) \neq P_{obj}(a,b)$, will occur depending on the choice of S. Clearly if any choice of S is possible then we can reverse the preference relation of a and b. For example, suppose, a =4 and b=5, so $P_{obj}(a,b)$ returns b. If we choose S_a={1,2,3} and S_b={6,7,8} then $f(a,S_a)$=3, and $f(b,S_b)$=0, so $P_{subj}(a,b)=a$. The subjective and objective preferences give opposite answers.

Even if S is the same for both a and b we can get an erroneous result. Consider, $S_a=S_b$={1,2,3}. Both a and b score the same as each other because they win against all opponents. Alternatively, we can choose S so that they lose against all opponents. So, $P_{subj}(a,b)$="draw". This corresponds to the 'loss of gradient problem' we described in the introduction.

To be sure, in a coevolutionary set-up, the composition of S is not arbitrary. But, we must be aware that even though the choice of a coevolutionary game may not seem problematic, we have already disconnected from the objective measure of performance. We will see that even our simple 'greater than' game can cause problems even in a quite normal coevolutionary set-up. Whereas, in an applied coevolution, the absence of an objective metric can prevent us from examining what is really happening, here we are able to illustrate these concepts clearly because we have access to both the subjective and objective fitnesses.

2.3 MULTIPLE DIMENSIONS

The second feature of our minimal substrate is the introduction of additional dimensions to the definition of an individual. That is, we may represent individuals by pairs of scalars, or vectors. For simplicity, let us discuss pairs, and call the two dimensions **x** and **y**. We will let each dimension represent a different aspect of a player's abilities. It is important to realize that we cannot necessarily reduce multiple dimensions to a single scalar value that will represent a player's quality. We cannot let the fitness of a player be represented by some weighted sum of its component dimensions, for example. This is because the value of the weighting for an aspect of play may depend on who the opponent is; for one opponent, **x** may be more important than **y**, for another opponent maybe only **y** is important. This subjectivity prevents us from reducing a multi-dimensional player to a single scalar and then determining a winner by comparing these values.

A simple way to model these aspects of coevolution is to allow some comparison between individuals to determine a single dimension that will, for these individuals, determine the outcome of the match. One way to do this is to choose that dimension in which the two players are most distinct. We define $f2(a,S)$, where a and each member of S are pairs, as follows:

$$f2(a,S) = \sum_{i=1}^{|S|} score2(a,S_i) \qquad \textbf{/eq.2}$$

where

$$score2((a_{\mathbf{x}},a_{\mathbf{y}}),(b_{\mathbf{x}},b_{\mathbf{y}})) = \begin{cases} score(a_{\mathbf{x}},b_{\mathbf{x}}), \text{ if } (|a_{\mathbf{x}}-b_{\mathbf{x}}|>|a_{\mathbf{y}}-b_{\mathbf{y}}|) \\ score(a_{\mathbf{y}},b_{\mathbf{y}}), \text{ otherwise.} \end{cases}$$

and, as before, $score(a,b)$= 1 if $a>b$, 0 otherwise.

[1] Notice that neither of these correspond to the Darwinian meaning of fitness relating to the number of viable offspring. Even in regular evolution the number of offspring an individual produces is regulated by the objective fitness of other individuals in the population as well as its own objective fitness.

This game is easily extended to more than two dimensions by asserting that whichever dimension has the biggest difference between opponents is the dimension that determines the outcome of the game. Note that the game has the desirable property that a generalist, a player that is maximal in all dimensions, can be defined that beats all other players. Accordingly, we assert that the objective of this game is to maximize all dimensions – i.e. the objective fitness of an individual is the sum of all dimensions. Potentially, a coevolutionary set-up could enable selective pressure to move from one dimension to the other dimension focusing on whichever is weakest. On the other hand, it might focus on one dimension to the detriment of other dimensions. We will use this game to model the effects of focusing and over-specializing that can occur in coevolution.

2.4 INTRANSITIVE SUPERIORITY

In Section 2.2, we considered the case where coevolution is erroneous in determining the superiority of two individuals when each is compared to some other sample of individuals. However, when using coevolutionary games, it is possible to create problematic scenarios even when comparing individuals against each other.

For example, it is quite conceivable that for three chess players, *A*, *B* and *C*, *A* can reliably beat *B*, *B* can reliably beat *C*, but *A* cannot beat *C*. Simply stated, we may say that the superiority of players in chess is not transitive. Further, suppose that *A* may be beaten by *C* creating a loop as in the "rock, scissors, paper" game – we might call this a game with *circular superiority relations* or *circular dominance relations*. This may result in local superiority relationships that provide a deceptive gradient and encourage strategies that are inferior in a global sense (e.g. further away from some strategy *D* which beats *A*, *B* and *C*). Or, coupled with over-specialization, coevolving species may drive each other from strategy to strategy, apparently improving, only to arrive back where they started.

The concept of intransitive superiority is central to issues in coevolutionary failure (Cliff & Miller 1995), and we want to be able to include it in our minimal substrate. To do this we will have to use at least a two dimensional game. Consider: if all the relevant characteristics of a player can be represented by a single value - for example, the ability of a javelin thrower can be characterized by distance alone - then such circular dominance is not possible. But in fencing, for example, the ability of a player is multi-dimensional including for example, the ability to parry, the ability to thrust, and stamina. As already stated, we cannot simply sum the ability of the swordsman in each of these respects - which of these characteristics is critical, or the weighting of these characteristics, depends on the characteristics of their opponent. In such cases where the ability of a player is multi-dimensional it is quite possible that three or more players may form a circular superiority relation.

A simple way to modify our game to incorporate intransitive superiority is to modify Equation 2 so that the dimension that determines the outcome of a game is the dimension in which the players are most similar (instead of most different). That is, when two players, $(a_{\mathbf{x}},a_{\mathbf{y}})$ and $(b_{\mathbf{x}},b_{\mathbf{y}})$, enter a game the winner will be whoever is the greater in the dimension in which they are closest.

$$f3(a,S)=\sum_{i=1}^{|S|} score3(a,S_i) \qquad \textbf{/eq.3}$$

where

$$score3((a_{\mathbf{x}},a_{\mathbf{y}}),(b_{\mathbf{x}},b_{\mathbf{y}}))=\begin{cases} score(a_{\mathbf{x}},b_{\mathbf{x}}), \text{ if } (|a_{\mathbf{x}}-b_{\mathbf{x}}|<|a_{\mathbf{y}}-b_{\mathbf{y}}|) \\ score(a_{\mathbf{y}},b_{\mathbf{y}}), \text{ otherwise.} \end{cases}$$

and, as before, $score(a,b)=1$ if $a>b$, 0 otherwise.

Using this game we can easily define three players that exhibit circular superiority, *a* beats *b* beats *c* beats *a*. For example, $a=(1,6)$, $b=(4,5)$, $c=(2,4)$: *a* beats *b* because they are closest in the **y** dimension and $a_y>b_y$; *b* beats *c* because they are closest in the **y** dimension and $b_y>c_y$, but *c* beats *a* because they are closest in the **x** dimension and $c_x>a_x$.

Note that this game still has the desirable property that a player that is maximal in both dimensions beats all other players. Again, we assert that the objective fitness of an individual in this coevolutionary game is the sum of all dimensions.

3 EXPERIMENTAL SET-UP

The following experiments use the games defined in Equations 1 through 3. In addition to defining the game we will use there are several other choices to be made in the set-up of the coevolution:

- Number of populations (who competes with who?, who reproduces with who?)
- Choosing members to make *S* (who plays who?)
- Sample size (how many do you play?)
- Selection scheme
- Variation operators

The following experiments will use one or two separate populations. Selection and reproduction in one population will operate independently of the other population in the cases where there are two. Unless otherwise stated, the population size is 25 for each population. In principle, the choice of who plays who is independent from the segregation of reproduction. However, in the following experiments when there is more than one population we shall limit ourselves to considering the case where players only play against opponents from the other population. Unless otherwise stated the sample size, $S=15$. We use fitness proportionate selection, and for simplicity we use mutation as the only variational operator. One detail we found illuminating concerns the bias of the mutation operator.

3.1 MUTATION BIASES

Because we are using such a simple substrate we must be careful about the assumptions we make with respect to the likelihood of beneficial and detrimental variations. If we imagine that our individuals are represented by real numbers then we might reasonably assume that a mutation would be equally likely to increase or decrease the value - perhaps we would add a random value drawn from a Gaussian distribution. If, alternatively, we were to represent individuals using a unary representation (simply the unitation, number of ones, of a fixed-length binary string) and vary values by mutating bits then mutation would have inherent biases. Specifically, a string with more than half zeros is more likely to increase than decrease, a string with more than half ones is more likely to decrease than increase, and in general, there is a natural bias towards strings with 50:50 ones and zeros.

In real applications, for example, a neural network controller, sorting networks, or a genetic programming game player, there are likely to be significant mutational biases. It may well be the case that a random neural network controller, sorting network or genetic program is likely to be superior to a null or default representation that might be used to initialize individuals; for example, a neural network with no connections or weights of 0, a sorting network with no comparitors, or a GP tree with no nodes. However, once a moderate solution has been found we would reasonably expect the situation to change. In the later stages of evolution it is likely to be the case that nearly all changes to an individual will be detrimental. We will call this situation a *negative mutation bias*. These basic observations have theoretic underpinnings in the simple models used by Fisher (1930).

Since we are abstracting an evolutionary substrate to a scalar (or two) we must be careful with assumptions like unbiased mutation. The following experiments will use a biased mutation. A simple way to do this is to evolve integers as if they were represented with a fixed length binary string and the value they represent is given by the unitation of the string. The (simulated) string length will be 100 and mutation per bit will be 0.05 probability of assigning a new random value.

4 EXPERIMENTS AND RESULTS

We start with a control experiment, and then conduct several experiments using Equations 1 through 3 to illustrate a few of the concepts we have discussed.

4.1 CONTROL: MUTATION BIAS

The first experiment is a control experiment using $f(a)=0$ to illustrate the effects of mutation bias and provide reference performance levels for the following experiments. We evolve single integer values in two separate populations with the biased mutation discussed previously. All individuals in the first population are initialized to 0. All individuals in the second population are initialized to 100. Figure 1 shows the populations evolving over time. The vertical axis represents the objective fitness of individuals. Reference lines are included at 50 and 100. The horizontal axis runs from generation 0 to generation 600.

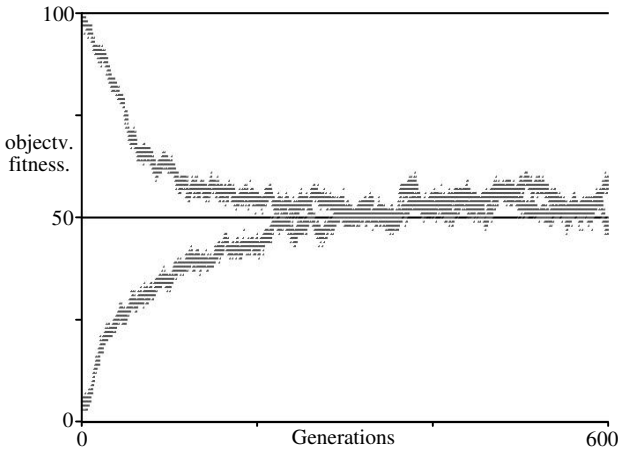

Figure 1: Neutral selection showing mutation bias.

We see that the population average is drawn to about 50 in both cases, as predicted. The performance level in the latter half of the run represents a neutral, no-selection, performance level for the populations.

4.2 EXPERIMENT 1: LOSS OF GRADIENT

Next we demonstrate that the subjective measure of fitness does not always deliver the desired objective performance even in the simple one-dimensional game of Equation 1. Figure 2 shows the performance of individuals in two coevolving populations.

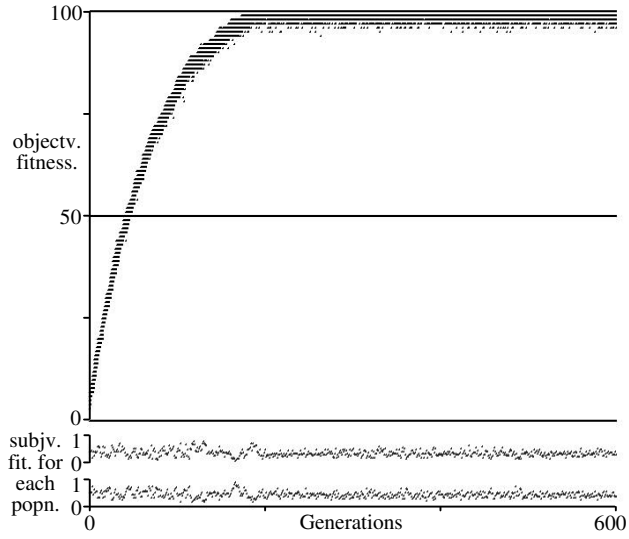

Figure 2: Coevolution using Equation 1.

The additional two plots in the lower section of the figure show the average subjective fitness for members of each population. These show that the subjective fitness of one

population is approximately one minus the subjective fitness of the other. They also show that the subjective fitness of either population does not vary significantly as evolution progresses despite the fact that objective fitness has changed. This is the Red Queen effect (Cliff & Miller 1995) – though the performance of individuals improves, the performance of their opponents improves at the same rate, and they find themselves no better off (subjectively). In an experiment where we do not have an objective measure of performance, this creates a problem for monitoring progress.

Figure 3 shows the same experiment but with a sample size, S=1. i.e. each player is evaluated by playing against one randomly selected player from the other population.

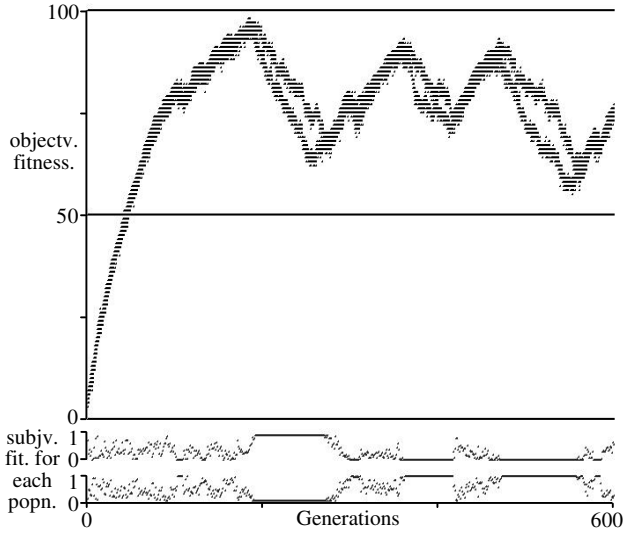

Figure 3: Coevolution using Equation 1, S=1.

This is clearly a different result altogether. There are clear downward trends as well as upward trends. Notice that the subjective fitnesses (at the bottom of the figure) show periods of polarization – one population scores 1 and the other population scores 0 – and these periods coincide with the downward trends in objective fitness. At these times all the individuals in the first population beat all the individuals they are tested against in the second population, or vice versa. This separation of the populations can be seen in the points plotted for the objective fitness values. Thus there is no selective pressure and the negative mutation bias is thus allowed to pull the population back down towards the neutral performance position shown in Figure 1. Then, by chance, the two populations happen to re-engage and race each other to high values again. This may happen repeatedly in a run.

In this game, the effect is only seen at these low population sizes and low sample sizes, and the good performance seen in Figure 2 can be regained using a larger population size, even with S=1. However, it is surprising that such a disconnection of the populations is possible at all in such a simple symmetric game. In a practical application of coevolution the likelihood of one population dominating the other may be affected by asymmetry in the game – for example, evasion may be easier than pursuit, and a population of evaders may get a little too far ahead on occasions, and cease to provide selective pressure. But note that even if the coevolving populations do not disconnect completely as they do here, the subjective fitnesses may be distorted.

4.3 EXPERIMENT 2: FOCUSSING

In these experiments we use Equation 2 to illustrate problems of focusing. We have already seen two populations coevolving successfully on a single dimension in Figure 2. Figure 4 shows two populations evolving on ten dimensions. To avoid using a larger genome, that would suffer unfairly from our mutation bias, we use ten dimensions of 10 bits each (instead of one dimension of 100 bits). The vertical axis shows the objective fitness of each individual, i.e. the sum over all dimensions.

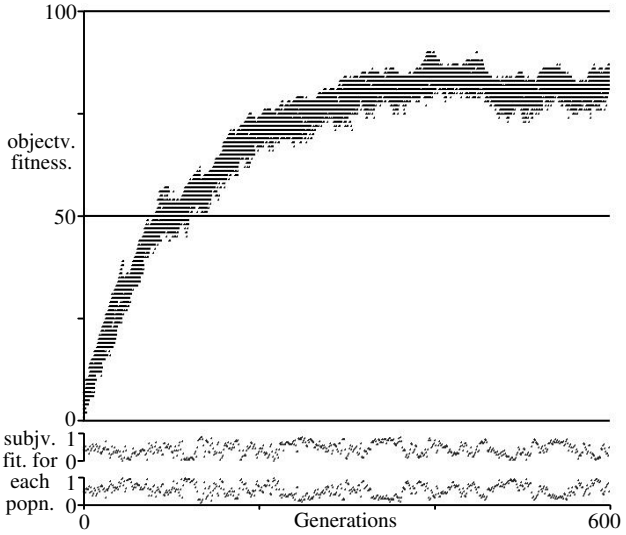

Figure 4: Coevolution using Equation 2, 10 dimensions.

Notice that the performance levels fail to reach 100. This can be explained by noticing that whilst any one dimension is the dimension that matters given the make-up of the other population, the other nine dimensions are likely to drift toward their neutral position. Although selective pressure switches from one dimension to another, high performance cannot be maintained in all dimensions simultaneously. This may cause individuals to 'forget' skills that they had learned previously, only to re-discover them later.[2] If our objective metric was concerned with only a subset of the ten dimensions then oscillations in performance would be pronounced. But, even when the objective metric values all dimensions equally we see that over-specializing can prevent the discovery of a generalist. Depressed performance also occurs in single-population coevolution using this game (drawing S from other members of the population).

[2] This is clear in these experiments when the performance in each dimension is observed separately (not shown).

In a normal evolutionary set-up, the failure to reach the maximum performance could be remedied with the use of elitism in the objective metric. But, note that elitism acting in the subjective metric cannot assist us here. We only have access to the 'apparent best' and elitism in this metric will not produce elitism in the actual (objective) best. However, a "Hall of Fame" method, where individuals play against representatives from past generations may decrease 'forgetting' and increase generalization (Cliff & Miller 1995).

4.4 EXPERIMENT 3: RELATIVISM

In our third experiment we examine the game in Equation 3 that exhibits intransitive superiority. Figure 5 shows the intransitive game with two populations.

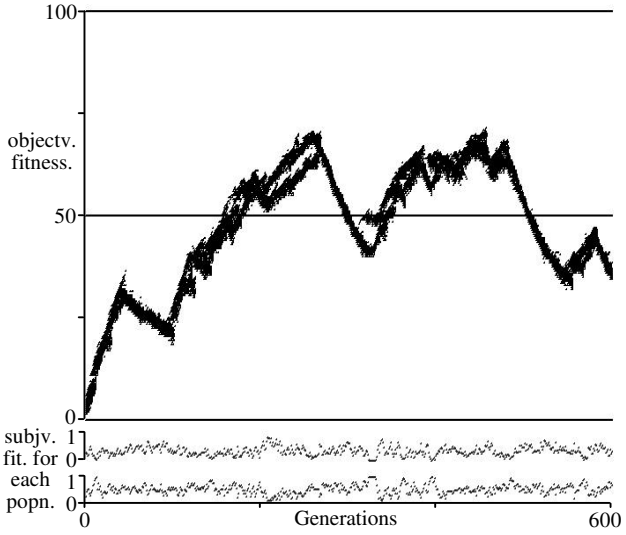

Figure 5: Coevolution using Equation 3.

Clearly, things are not working the way we want them to according to our objective metric. Notice that the downward trends are not accompanied by a domination of one population by the other – the average subjective fitnesses are not polarized as they are in Figure 3. So the downward trends are not the result of drifting under negative mutation bias. Also, the downward excursions sometimes go below the neutral level of 50 showing that the populations are actually being driven downwards.

This activity can be explained by noticing that subjective scores in this game can sometimes be improved by lowering the value a player represents. Specifically, if a player is losing in the chosen dimension it may be possible to change which dimension is relevant by lowering its value. In some circumstances, this may make the second dimension become relevant and the outcome of the game may be different. For example, consider a=(4,7) and b=(5,5). The closest dimension is the first, and b wins. Now, a'=(3,6) is a small variation from a. The closest dimension when a' plays b is the second dimension and a' wins. So, a' is preferred over a even though a' is inferior to a in the objective metric.

So, whereas Figure 3 showed how subjective preference may give a draw where objective preference should give a winner, in this experiment, we see that P_{subj} may give the opposite answer to P_{obj}. As a result, we see that performance can be *driven* down not just *drift* down. This dynamic is produced by the exact characteristics of the game we defined. However, it is sufficient to illustrate the point that P_{subj} can be the reverse of P_{obj} even in a game which looks innocent enough. The difficulty that Equation 3 causes arises from the fact that the features of a player that control a win with respect to one player, are in opposition to the features that will win against another player. Specifically, reduction in some dimension can allow a win against one player, whilst inducing a loss against another player. We may expect such destructive dynamics in any game with these counter acting properties.

Figure 6 shows that the effects of intransitive superiority can be destructive even in one-population coevolution. That is, even when players play against opponents from their own population, the intransitive nature of the game can prevent continued increases in performance. Interestingly, the data from this run can be seen to exhibit some 'spontaneous speciation'. Although there is only one population, the individuals occasionally diverge showing two separate sub-populations.

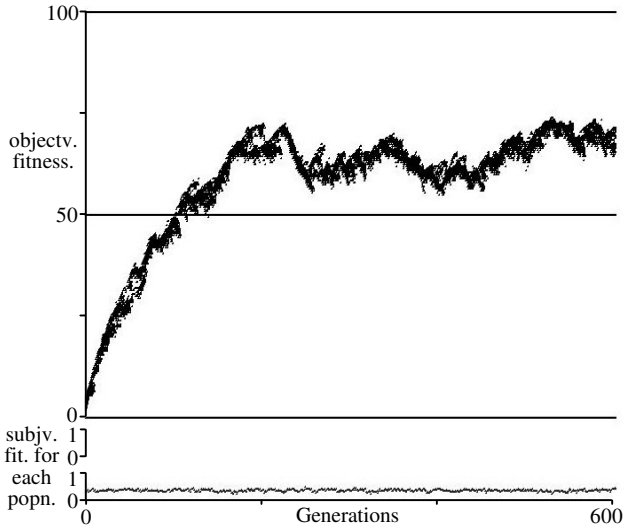

Figure 6: Coevolution using Equation 3: one population.

The phenomena in these experiments with Equation 3 are not overcome by larger population sizes and larger sample sizes. Examining the exact values in both dimensions (not shown in these figures) reveals that evolution in this game is indeed moving through the same parts of strategy space repeatedly. Thus this simple game illustrates the cyclic activity often speculated about in coevolution literature (Cliff & Miller 1995).

5 CONCLUSIONS

With the use of our minimal substrate we have provided concrete illustrations of several coevolutionary issues. We have given simple examples in which subjective fitness measures appear unproblematic but can actually disagree with objective fitness.

We illustrated three kinds of coevolutionary failure: *loss of gradient* where performance drifts (downward) because one population dominates the other, *over-specialization* where coevolution fails to find general solutions because strategies transition from one dimension to the other exploiting specific weaknesses, and issues of *relativism* where subjective fitness can act in opposition to objective fitness. Each of these weaknesses in coevolution can cause repeated 'forgetting' and re-discovery of strategies and prevent the continued improvement in performance that we would like to see.

Important concepts in these illustrations include the separation of *objective* and *subjective* fitness: the metric that we as researchers seek to optimize, and the metric of performance as perceived by the co-evolving individual, respectively. Also, the fact that a coevolutionary game may not be reducible to a single dimension – the performance of an individual is always *with respect to* some other individual (or set of individuals) – thus the subjective metric may not be reduced to a one-dimensional notion of quality, or a single superiority ordering. Finally, intransitivity is an important characteristic of subjective superiority that can be particularly problematic.

In illustrating these problems and concepts we have made many choices both in the game and the coevolutionary set-up. Our substrate is by no means the only simple substrate in which these concepts could be illustrated. Nonetheless, the coevolution of scalars and vectors provides one concrete example for several of the slippery issues common in the coevolution literature. And, unlike previous work, in this substrate we are able to properly separate the issues of coevolution from the issues of any complex application domain. The problems caused by these simple games caution us in making assumptions about more complex coevolutionary endeavors.

In previous work we have used the term 'mediocre stable state' to mean what we may now describe as a condition where the coevolutionary system is not producing improved performance in the objective metric despite continued adaptive steps in the subjective metric. This paper has begun to decompose the mechanisms that may be behind such failures, and in so doing, it may assist us in at least diagnosing problems in future. Related work builds upon the insights here to formulate an optimization method that explicitly respects the multi-dimensional nature of coevolutionary games by applying the notions of multi-objective optimization to a set of subjective scores.

Acknowledgments

The first author would like to thank the (past and present) members of DEMO at Brandeis, notably Alan Blair, Hughes Juilles, Paul Darwen, and Greg Hornby, but especially, Sevan Ficici for numerous invaluable discussions about the nature of coevolutionary difficulty.

References

Cliff, D, & Miller, GF, 1995, "Tracking the Red Queen: Measurements of adaptive progress in co-evolutionary simulations", *Third European conference on Artificial Life*, pp 200-218, Springer-Verlag, LNCS 929.

Ficici, SG. & Pollack, JB., 1998, "Challenges in Coevolutionary Learning: Arms-Race Dynamics, Open-Endedness, and Mediocre Stable States." *Proceedings of the Sixth International Conference on Artificial Life*. Adami, et al, eds. Cambridge: MIT, Press.

Fisher, RA, 1930, *The genetical theory of natural selection,* Clarendon press, oxford.

Goldberg, DE, 1989 *Genetic Algorithms in Search, Optimization and Machine Learning*, Reading Massachusetts, Addison-Wesley.

Hillis, DW, 1992, "Coevolving parasites improve simulated evolution as an optimization procedure", In Langton, C, ed., *Artificial Life II*, Addison Wesley.

Holland, JH, 1975 *Adaptation in Natural and Artificial Systems*, Ann Arbor, MI: The Uni. of Michigan Press.

Juille, H, & Pollack, JB, 1996, "Coevolving intertwined spirals", *procs. of fifth annual conference on evolutionary programming*, pp461-468. MIT press.

Juillé, H. & Pollack, JB, 1998, "Coevolving the 'Ideal' Trainer: Application to the Discovery of Cellular Automata Rules." *Proceedings of the Third Annual Genetic Programming Conference*, Madison, WI, July.

Kauffman, S, 1993, *The Origins of Order*, Oxford University Press.

Maynard-Smith, J., 1982, *Evolution and the Theory of Games.* Cambridge University Press, Cambridge.

Miller GF, & Cliff D, 1994, "Protean Behavior in Dynamic Games: Arguments for the Co-Evolution of Pursuit-Evasion Tactics" in Cliff, D, Husbands, P, Meyer, JA, and Wilson, WS, eds. *From Animals to Animats 3: Procs. of Third International Conference on Simulation of Adaptive Behavior* (*SAB94).* MIT Press, pp.411-420.

Noble, J, & Watson, RA, "Pareto Coevolution: Using performance against coevolved individuals as dimensions for pareto selection", Spector, L, et al, editors. GECCO-2001: Proceedings of the Genetic and Evolutionary Computation Conference. Morgan Kaufmann, 2001.

Pollack, JB, Blair, A. & Land, M. 1996, Coevolution of A Backgammon Player. *Proceedings Artificial Life V*, C. Langton, (Ed), MIT Press.

Reynolds, CW, 1994, Competition, Coevolution and the Game of Tag, in the *proceedings of Artificial Life IV*, R. Brooks and P. Maes, Editors, MIT Press, Cambridge, Massachusetts, pp 59-69.

Sims, K, 1994, "Evolving 3d morphology and behavior by competition", in Brooks & Maes, eds., *Artificial Life IV*, pp28-39, MIT press.