Solution Concepts and Monotonicity

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Desired Algorithm Operation

• Assume our coevolutionary algorithm never discards a strategy once discovered
• If we query the algorithm (at “appropriate” points) over time, we desire the solution returned at time $t+1$ to be no worse than that at time $t$
• We desire monotonic improvement
• (Elitism in ordinary EA does this)
• In coevolution, whether we get monotonic improvement depends on the solution concept used
1) illegal strategies
2) solution may be unrepresented
3) features of 1 and 2
4) robot locomotion and tic-tac-toe
Measurement of Behavior

- Payoff indicates success of behavior w.r.t. a single metric of behavior; higher is better
- Payoffs are not utilities; utilities assume common currency of goodness
• Each behavior is a *pure strategy* of the game, but may be mixed strategy of the domain.
Game Types

- **Single-player game against nature**
  - Behaviors: \( w, x, y, z \)
  - Metrics: \( s, d, a \)
  - States of nature: \( N1, N2, N3 \)
    - Behaviors: \( sda, sda, sda \)

- **Multi-player game**
  - Behaviors: \( w, x, y, z \)
Sub-Game
Solution Concept

- Solution concept integrates over multiple metrics of goodness to give a holistic assessment of quality

domain → substrate → game → solution concept → solution(s)

what we care about → what we observe → metrics → we apply
Configuration and Solution

• Given $n$-player game, a configuration represents players’ strategy choices:
  \[ K = < s_1, s_2, ..., s_n > \] (note: $s_i$ may be a set)
• According to solution concept, some configurations are solutions, denoted $K^*$
• Solution set is set of solutions for game $G$ according to solution concept $O$:
  \[ S^*(G, O) = \{ K^* | G, O \} \]
Preference Relation I

- For a given game G, we prefer a solution $K^*$ to a non-solution (configuration) $K$.
- We prefer configuration $K_\alpha$ to $K_\beta$ iff:
  
  for each subgame $G_\beta$ where $K_\beta$ is the solution, there exists a game $G_\alpha$ where $K_\alpha$ is the solution, and $G_\alpha \supset G_\beta$. 

![Diagram showing preference relations between subgames](attachment:diagram.png)
Preference Relation II

• Preference relation is transitive, asymmetric, ~reflexive

• Preference relation gives us a *poset* of configurations
Monotonicity

- We prefer $K_x$ to $K_y$, yet $E$ is a subgame of $D$.
- Given subgames $\alpha$, $\beta$, $\gamma$, where $\alpha \supseteq \beta \supseteq \gamma$.
- If $K$ is solution to $\alpha$ and $\gamma$, but not $\beta$, then solution concept is not monotonic; if all three, then monotonic.

Subgames where $K_x$ is a solution

Subgames where $K_y$ is a solution
Solution Concepts & Monotonicity

• Monotonic concept guarantees that solution improves monotonically with time, when strategies are never discarded
• Nash equilibrium is a monotonic concept
• Non-dominated front is monotonic only if identical appearing strategies are not allowed on front
• “Best-scoring strategy” is not monotonic
Sub-games and their solutions for Nash concept from a 5-strategy game
Monotonicity and Open-Endedness

• Open-ended arms race requires:
  – proper substrate (working on this...)
  – proper algorithm (we’ve learned a lot about this)
  – monotonic solution concept

• Because of monotonicity, we will never return to a configuration once we leave it; we will never return to offspring configurations
Same 5-strategy game, non-domination concept (non-monotonic version)
Random seven-strategy games
Random nine-strategy games