Intrinsic Geometry of Coevolutionary Domains

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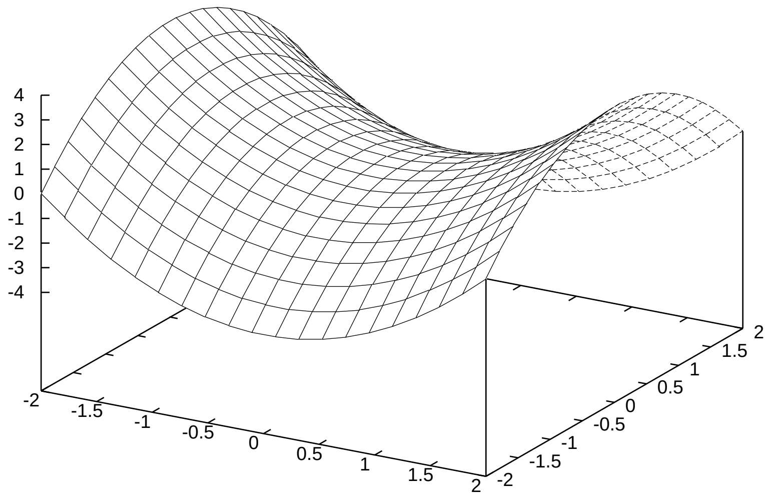
The Fitness Landscape

Traditionally, the fitness landscape is how geometry entered into evolutionary computation. Michael Conrad's 1990 *The Geometry of Evolution* was very influential.

The idea: treat the genotype as a spatial structure, usually a graph (Terry Jones) or a topology (Peter Stadler). Then the objective function $f : S \to \mathbb{R}$ is a surface over that space.

Concepts like *fitness saddles*, *ridges*, and *local optima* come directly from this conceptualization of objective-function-as-surface.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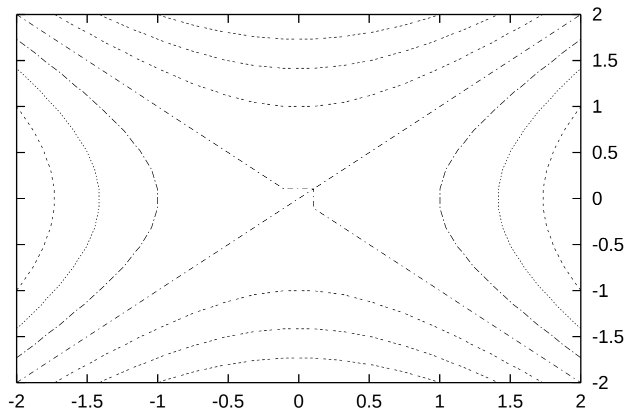
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Coevolution Requires an Alternative

Coevolution does not fit happily into this picture. Fitness landscape should be independent of the current population; we want the population to be *navigating* the landscape.

Attempts to view coevolution as navigating through a changing landscape have not been enlightening. There are too many degrees of freedom in the population to get our heads around how this looks.

What we want is a simple, static space associated with a problem. We want to view an algorithm as *navigating* this space. A big question is, how could we do that in coevolution?

Intrinsic Structure of a Game

"Coevolutionary statics" (Bucci and Pollack 2002) refers to a domain's structure, independent of representation. This paper suggested an alternative to the fitness landscape which would be appropriate for coevolution.

The idea: represent a domain as a function. For simplicity, let's consider domains like board games which can be represented with a function:

$$p : S \times S \to \{0 < 1\}$$

Then p is exactly the incidence matrix of a directed graph structure on S.

Intrinsic Structure of a Game, 2

We can view a directed graph as a sort of topological space. There are many ways to do that, but here's a new one for EC:

1. Find a "non-degenerate preorder cover" for the graph;
2. Embed each preorder into $\mathbb{R}^n$ (Bucci and Pollack 2003);
3. Cut out the hull of each preorder;
4. Glue the hulls together according to the "recipe" of the graph.

The result is an n-dimensional spatial representation of the original graph, into which the graph embeds.

Examples

1. rock-paper-scissors and n-cycles
2. fractal rock-paper-scissors
3. LINT
4. MOO (vs. Stadler and Flamm)

Uses

1. Alternative concepts to “local optima” like “extended dimension” and “circular dimension;”
2. View of genotype’s role as facilitating navigation
3. Cataloging games via family resemblance of their spaces (LINT as cylinder, fractal games as toroids);
4. Visualizing algorithm behavior: plot population on game space.
5. Many search algorithm implications!

Discussion

This preorder cover decomposition of a graph is closely related to the previous decomposition I presented. Every graph has such a decomposition – the set of edges is a 1-d one. The higher-dimensional the terms of the decomposition, the more revealing it is.

Conclusions

We claim that to bring coevolution into the EC fold, a different geometric conceptualization of problems is necessary.

We suggest problems have intrinsic geometry and that representation is an agent for navigating that geometry. Ordinary EC mixes these up. Issues of search are displayed in the geometric structure and in the navigation strategy.

We present a way of elucidating the geometric structure.

Future Work

1. *Invariance proof*: show that the spatial qualities we care about are invariant to the arbitrary choices of a preorder cover;

2. *Efficient Algorithm*: brute force is clear, but how to do this efficiently?

3. *Visualization*: how to hook this to a visualizer like Mathematica.